Chapter 2
Section 2.4

Activity

A college student is hired to deliver new telephone books and collect the old ones for recycling. She is paid \$6 per hour plus \$ .30 for each old phone book she collects. Her salary is a function of the number of phone books collected. However, she must use her own car for this job. She figures that her car expenses average \$0.20 per phone book collected plus a fixed cost of \$20 a week for insurance. We can write her expenses as a function of phone books collected as well. Her profit for one week is her salary minus her expenses. So we can also write her profit as a function of phones books collected!

a) If x is the number of phone books she collected, then what is her salary as a function of x.

S(x)= $6(40)+0.30x = 240 + 0.3x$

b) Write her expenses as a function of phone books collected.

E(x)= $20 + 0.2x$

c) Write her profit as a function of phone books collected.

P(x)= $S(x) - E(x) = (240-20) + (0.3-0.2)x = 220 + 0.1x$

Basic Operations with Functions

Definition: For two functions f and g, the **sum, difference, product** and **quotient functions**, are the functions $f+g, f-g, f\cdot g$, and f/g, respectively, and are defined as follows:

$$(f+g)(x) = f(x)+g(x)$$
$$(f-g)(x) = f(x)-g(x)$$
$$(f\cdot g)(x) = f(x)\cdot g(x)$$
$$(f/g)(x) = f(x)/g(x) \quad \text{provided that } g(x)\neq 0.$$

Exercise: Let $h=\{(1,3),(2,8),(3,6),(5,9)\}$ and $f=\{(1,6),(2,11),(3,0),(4,1)\}$. Let $j(x)=\sqrt{x}$ and $g(x)=3x-1$.**Find each function and state its domain.**

a) $h+f$ $\quad dom(h+f)=\{1,2,3\}$ $\quad (h+f)(1)=3+6=9$, $(h+f)(2)=19$, $(h+f)(3)=6$ $\quad h+f=\{(1,9),(2,19),(3,6)\}$

b) $h\cdot f$

c) h/f

d) $j+g$

e) $j\cdot g$

f) j/g

Composition of Functions

Definition: If f and g are two functions, the **composition** of f and g, written $f \circ g$, is defined by the equation

$$(f \circ g)(x) = f(g(x)),$$

provided that $g(x)$ is in the domain of f. The composition of g and f, written $g \circ f$. is defined by

$$(g \circ f)(x) = g(f(x)),$$

provided that $f(x)$ is in the domain of g.

Exercise: Let $f(x) = \sqrt{x}, g(x) = 2x - 1$ and $h(x) = x^2$. **Find each composition and state its domain.**

a) $f \circ g$

b) $g \circ f$

c) $h \circ f$

d) $h \circ g \circ f$

e) $f \circ g \circ h$

Question: Is it true or false that $f \circ g = g \circ f$? False

Exercise: Let $f(x) = \sqrt{x}, g(x) = x - 3$ and $h(x) = 2x$. Write each given function as a composition of appropriate functions chosen from f, g and h.

a) $F(x) = \sqrt{x-3}$ $F(x) = f \circ g(x)$

b) $G(x) = x - 6$ $G(x) = g \circ g(x)$

c) $H(x) = 2\sqrt{x} - 3$ $H(x) = g \circ h \circ f(x)$

Composition with Formulas

Exercise: The radius of a circle is a function of the diameter ($r = d/2$) and the area is a function of the radius ($A = \pi r^2$). Construct a formula that expresses the area as a function of the diameter.

$A = \pi\left(\frac{d}{2}\right)^2$

Exercise: A student's salary (in dollars) for collecting x phone books is given by $S(x) = 0.30x + 240$. The amount of withholding (for taxes) is given by $W(x) = 0.20x$, where x is the salary. Express the withholding as a function of the number of phone books collected.

$$W \circ S(x) = W(S(x)) = 0.20 \cdot S(x)$$
$$= 0.20(0.30x + 240)$$
$$= 0.06x + 48$$